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Superfluid density of ^{4}He near T_λ in two-loop order

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We present the results of a renormalization group calculation of the superfluid density of ^{4}He near T_λ at finite superfluid velocity $\mathbf{v}_s$. Within the ϕ^4 model for an order parameter with O(n) symmetry and using field theory in three dimensions, we have computed to two-loop order the amplitude function for the superfluid density up to $O((v_s/v_{sc})^2)$. We also give the two-loop expression for the amplitude function of the order parameter at $\mathbf{v}_s = 0$. Applications of these results are briefly discussed.

1. Introduction

The superfluid density ρ_s is a quantity of fundamental relevance to the properties of ^{4}He below the λ-transition. These include not only equilibrium critical properties but also phenomena at finite counterflow velocity $\mathbf{w} = \mathbf{v}_s - \mathbf{v}_n$.

The λ-transition in ^{4}He at $\mathbf{w}=0$ is ideal for testing the universality predictions of the renormalization group (RG) theory of critical phenomena [1]. Such a test consists not only in comparing the measured values for exponents and amplitude ratios with theory but also in demonstrating that these values are independent of pressure. To avoid gravity-induced nonuniversal effects, high-precision measurements of the superfluid fraction ρ_s/ρ in a microgravity environment are planned for future research [2]. Correspondingly, on the theoretical side, it is necessary to compute the asymptotic and nonasymptotic critical behavior of ρ_s as accurately as possible.

For the case $\mathbf{w} \neq 0$, knowledge of the $\mathbf{w}$-dependence of $\rho_s(\mathbf{w})$ is of crucial importance for the understanding of superfluid ^{4}He at finite heat current $Q = -ST\rho_s(\mathbf{w})\mathbf{w}$ [3]. In the following, we use the frame where $\mathbf{v}_n = 0$ and $\mathbf{w} = \mathbf{v}_s$.

2. Superfluid density at $\mathbf{v}_s = 0$

According to RG theory, the superfluid density can be expressed in the form [4]

$$\rho_s = \frac{k_B T}{4\pi}\left(\frac{m_4}{\hbar}\right)^2 \xi_-^{-1} G(u(l_-)) \qquad (1)$$

where $G(u)$ is the amplitude function, u is the renormalized coupling, ξ_- is an appropriately defined correlation length [4] and $l_-(t)$ is the flow parameter [$t = (T - T_\lambda)/T_\lambda$ is the reduced temperature]. An analogous expression exists also for the order parameter $\langle\phi\rangle^2$ with an amplitude function $f_\phi(u)$ [4].

The functions G and f_ϕ were determined within the O(n) symmetric ϕ^4 model and for the minimally renormalized theory in three dimensions in Ref. 4, but only in one-loop order. We have now calculated the coefficients of the two-loop [O(u)] terms for these quantities. The resulting two-loop expressions are

$$G = \frac{1}{8u} + \frac{1}{3} + \left[8(n-3)\ln 3 - \frac{(683n-2378)}{54}\right]u \qquad (2)$$

and

$$4\pi f_\phi = \frac{1}{8u} + \left[8(n-1)\ln 3 - \frac{(328n-640)}{27}\right]u, \qquad (3)$$

respectively. At an intermediate stage of the calculation, spurious singularities due to Goldstone modes appear, which we have regulated by the introduction of a small external field h. These divergent terms cancel among themselves so that the bare series for ρ_s and $\langle\phi\rangle^2$ remain finite as $h \to 0$.

The graph below shows $uG(u)$ as a function of u for $n = 2$ (corresponding to superfluid ^{4}He). At the fixed point $u(0) = u^*$, the one- and two-loop terms each contribute about 10% relative to the zero-loop term.

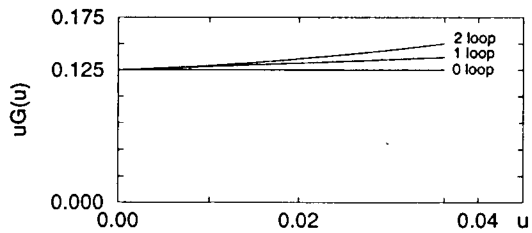

Figure 1: Amplitude function $uG(u)$ for the superfluid density in 0-, 1-, and 2-loop order. The lines terminate at the fixed point $u(0) = u^* = 0.0362$.

3. Superfluid density at $\mathbf{v}_s \neq 0$

The formula in (1) can be generalized to the situation in which there is a finite superfluid velocity $\mathbf{v}_s \neq 0$ [3]. For $v_s < v_{sc}$, corresponding to $\kappa < \kappa_c \simeq 0.4$, the superfluid density ρ_s is given by

$$\frac{\rho_s(\kappa)}{\rho_s(0)} = 1 + \frac{G^{(2)}(u(l_-))}{G(u(l_-))}\kappa^2 + O(\kappa^4) \qquad (4)$$

where

$$\kappa = \xi_- \left(\frac{m_4}{\hbar}\right) v_s \qquad (5)$$

and where $\rho_s(0)$ and $G(u)$ are given by (1) and (2), respectively. As for $G(u)$ above, we have computed the two-loop $[O(u)]$ term of $G^{(2)}(u)$. The result is [5]

$$G^{(2)}(u) = -\frac{1}{4u} + \frac{56}{15} + g_{22}u, \qquad (6)$$

for $n = 2$, where

$$g_{22} = \frac{2264}{45} - \frac{128}{3}\ln 3 - \frac{1024}{15}\ln 2. \qquad (7)$$

Unlike the calculation for $\rho_s(0)$, the coefficient of κ^2 in $\rho_s(\kappa)$ is not plagued by spurious Goldstone singularities when $n = 2$. In view of the application to ^{4}He and because of the considerable simplification involved in carrying out the calculation at $h \equiv 0$, we have therefore restricted ourselves to the case $n = 2$.

In Fig. 2 we plot $\rho_s(\kappa)/\rho_s(0)$, for $u(0) = u^*$, as a function of κ showing the effect of using the functions $G(u)$ and $G^{(2)}(u)$ in zero-, one- and two-loop order. Also shown is the full κ-dependence obtained in one-loop order by Haussmann and Dohm (see Fig. 5 of Ref. 3). The closeness of the one-loop curve obtained from (4) to the result of Ref. 3 for $\kappa \lesssim 0.75\kappa_c$ suggests the range in which the expansion with respect to κ^2 (4) is a good approximation. The small difference between the one- and two loop curves in this range gives further confidence to the low-order (in u) minimally renormalized perturbation approach in $d = 3$.

4. Applications

(i) The two-loop formula (2) for $G(u)$ can be used to obtain values for the universal amplitude ratios $\xi_0^T (A^-)^{1/3}$ and a_C^-/a_{ρ_s} where ξ_0^T is the amplitude of the transverse correlation length [4]. Evaluation of the amplitude ratios requires the fixed point values of various amplitude functions and their derivatives (in the case of correction ratios).

(ii) Experimental data for ρ_s/ρ and for the specific heat $C_P^\pm$ are usually analyzed by use of the asymptotic forms [1]

$$\frac{\rho_s}{\rho} = k_0|t|^\zeta(1 + a_{\rho_s}|t|^\Delta + \cdots), \qquad (8)$$

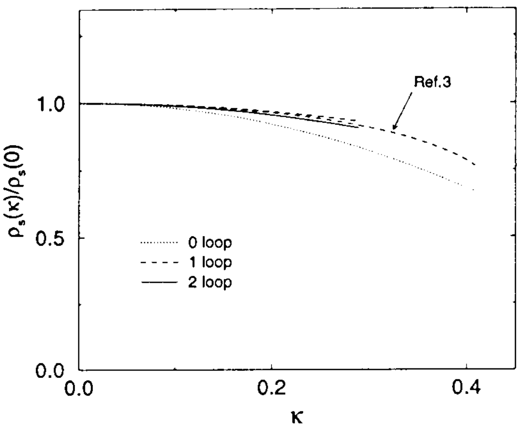

Figure 2: Superfluid density as a function of $\kappa = \xi_-(m_4/\hbar)v_s$ in the range $0 \leq \kappa \leq \kappa_c$ where $\kappa_c \simeq 0.4$ [3]. The curves obtained from (4) extend as far as $\kappa \simeq 0.75\kappa_c$.

$$C_P^\pm = B + \frac{A^\pm}{\alpha}|t|^{-\alpha}(1 + a_C^\pm|t|^\Delta + \cdots). \qquad (9)$$

It has been suggested [6] that, instead of using (8) and (9), one may analyze the data by use of the full RG formulas which, through amplitude functions like $G(u)$, contain the full information about the nonasymptotic behavior within the ϕ^4 model (at infinite cut-off). This has the advantage of including the *entire* Wegner series, of which the $O(|t|^\Delta)$ terms are just the leading contributions, without introducing additional nonuniversal parameters.

(iii) The superfluid density $\rho_s(\mathbf{v}_s)$ at finite $\mathbf{v}_s$ is needed to determine the superfluid current $\mathbf{J}_s = \rho_s(\mathbf{v}_s)\mathbf{v}_s$ which is related to the enhancement of the specific heat of superfluid ^{4}He in a heat current [7].

REFERENCES

[1] A. Singsaas and G. Ahlers, Phys. Rev. B **30** (1984) 5103.

[2] J. A. Lipa, V. Dohm, U. E. Israelsson and M. J. DiPirro, NASA Proposal, NRA 94-OLMSA-05 (1995).

[3] R. Haussmann and V. Dohm, Phys. Rev. B **46** (1992) 6361.

[4] R. Schloms and V. Dohm, Phys. Rev. B **42** (1990) 6142; Europhys. Lett. **3** (1987) 413.

[5] M. Strösser, Diplom Thesis, TH Aachen (1996).

[6] V. Dohm, J. Low Temp. Phys. **69** (1987) 51.

[7] R. Haussmann and V. Dohm, Phys. Rev. Lett. **72** (1994) 3060; and at this conference.