SCALING FUNCTIONS OF THE SPECIFIC HEAT OF CONFINED ^{4}HE NEAR T_λ *

Ulf Mohr, Michael Pitz, and Volker Dohm

Institut für Theoretische Physik, Technische Hochschule Aachen,

D–52056 Aachen, Germany

One of the fundamental predictions of the renormalization-group (RG) theory as applied to confined ^{4}He [1] is the finite-size scaling structure of the specific heat near T_λ. These predictions resulting from field-theoretic calculations are summarized as follows.

The (constant-pressure) specific heat $C(t, L)$ per unit volume of ^{4}He at the reduced temperature $t = (T - T_\lambda)/T_\lambda$ in a confining geometry with a characteristic length L has the asymptotic scaling structure

$$C(t, L) - C(t, \infty) = -(L/\xi_0)^{\alpha/\nu} f_2 \left(t(L/\xi_0)^{1/\nu}\right) \tag{1}$$

where $C(t, \infty)$ is the bulk specific heat and f_2 is a scaling function. This scaling function depends on the geometry and on the boundary conditions (b.c.) of the order parameter. The representation (1) is useful in the range of large $|t(L/\xi_0)^{1/\nu}|$ where $f_2(x) \sim (A_s^\pm/\xi_0)|x|^{-\alpha-\nu}$ represents the surface specific heat $C_s = A_s^\pm |t|^{-\alpha-\nu}$ for $t > 0$ and $t < 0$, respectively.

An alternative useful scaling representation is

$$C(t, L) - C(t_0, \infty) = (L/\xi_0)^{\alpha/\nu} f_1 \left(t(L/\xi_0)^{1/\nu}\right) \tag{2}$$

where $t_0 = (\xi_0/L)^{1/\nu}$ is the reduced temperature at which the correlation length ξ above T_λ is equal to L. The scaling function $f_1(x)$ is smooth and finite near $x = 0$ and has the advantage of characterizing the specific heat maximum better than the function $f_2(x)$.

So far there exist *quantitative* theoretical results for the scaling function $f_1(x)$ only for *cubic geometry* with *periodic* b.c. [2]. Relevant for ^{4}He near solid boundaries, however, are Dirichlet b.c. (vanishing order parameter at the boundaries) and non-cubic geometries. For the latter case the theory is not yet fully developed and only a few predictions for

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f_1 and f_2 exist [3-5]. Here we report on recent progress and on planned field-theoretic calculations of the surface specific heat C_s above and below T_λ. These calculations aim at a comparison with forthcoming data of the confined helium experiment (CHeX) to be performed under microgravity conditions [6].

Our theory is based on the statistical distribution $\exp\{-\mathcal{H}\}$ with the Landau-Ginzburg-Wilson functional

$$\mathcal{H} = \int d^3x \left[\frac{1}{2} r_0 |\psi|^2 + \frac{1}{2}|\nabla\psi|^2 + u_0|\psi|^4\right] \quad (3)$$

for the complex order parameter $\psi(\mathbf{x})$ (Bose condensate wave function) in a plate geometry with separation L.

There are three regimes of the scaling variable $x = t(L/\xi_0)^{1/\nu}$ which require different approaches: the regimes $|t|(L/\xi_0)^{1/\nu} \gg 1$ above and below T_λ where surface effects are dominant and the region $|t|(L/\xi_0)^{1/\nu} \lesssim 1$ including the specific heat maximum. In the surface regime *below* T_λ , a renormalized mean-field approach (RMFA) yields the leading contribution [7]. *Above* T_λ , the surface effect is due to fluctuations that can be calculated perturbatively in a loop-expansion [8]. The treatment of the maximum region poses problems, because in this region a continuum of modes has to be taken into account in the mode expansion for the semi-infinite plate geometry [1,4,5].

On a semiquantative level the expected temperature dependence of the specific heat $C(t, L)$ confined between plates of separation $L = 0.057$mm [6] is shown in Fig. 1 as a function of t. (The bulk curve (dashed line) is taken from the asymptotic representation given in [9] with $\alpha = -0.01285$ and $\nu = (2-\alpha)/3$.) This demonstrates the dominant role of the surface effects induced by the suppression of the order parameter at the confining plates.

Our results for $C(t, L)$ in the surface regime below T_λ (in RMFA) is shown in Fig. 1(a) (solid line for $t \lesssim -10^{-7}$) and the corresponding surface part of $f_2(x)$ is plotted in Fig. 2(a) as dashed line. A calculation of the leading fluctuation correction in half-space geometry is in progress [8].

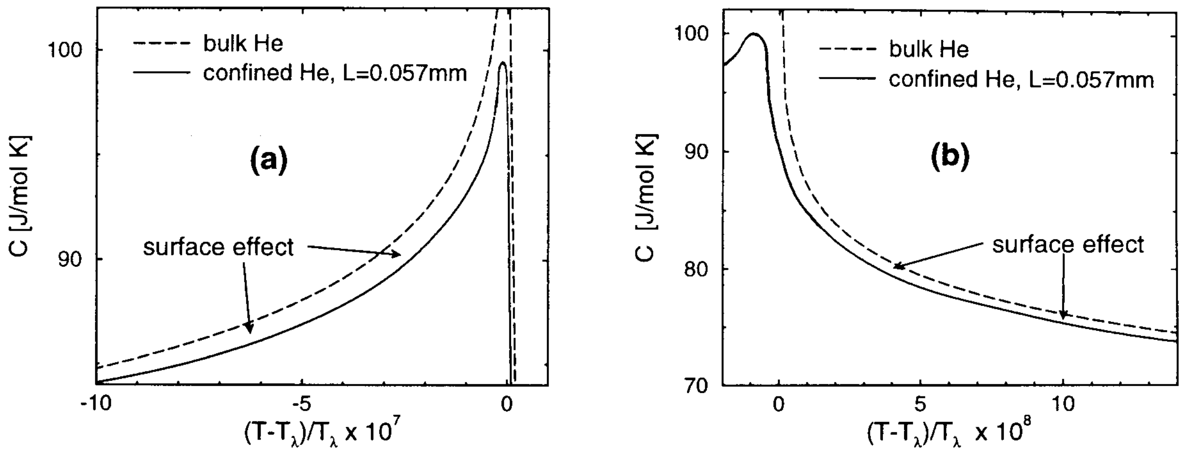

Fig.1: Expected temperature dependence of the specific heat for (a) $T \lesssim T_\lambda$ and (b) $T \gtrsim T_\lambda$.

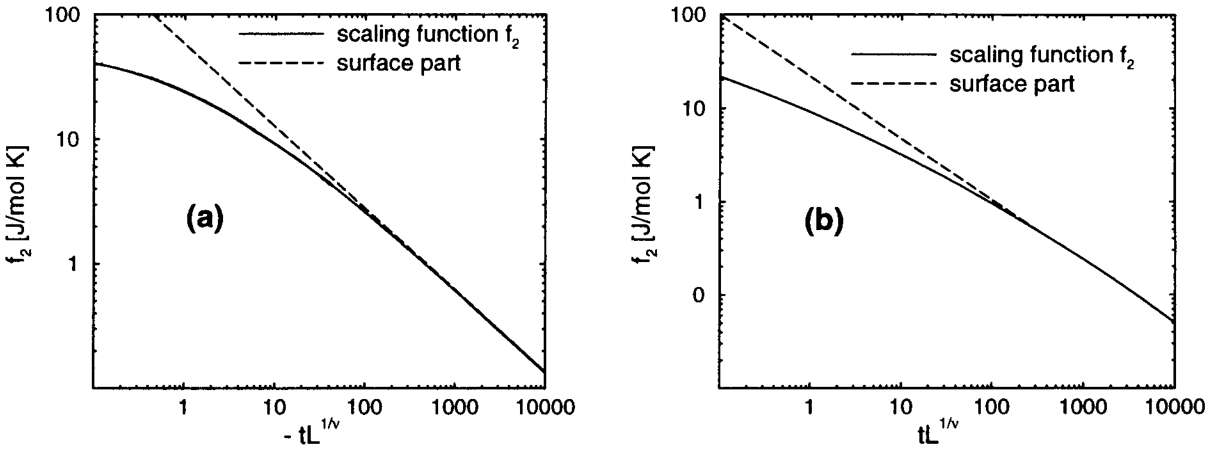

Fig.2: Expected shape of the scaling function f_2 vs $|t|\, L^{1/\nu}$ with L in Å for (a) $T < T_\lambda$ and (b) $T > T_\lambda$.

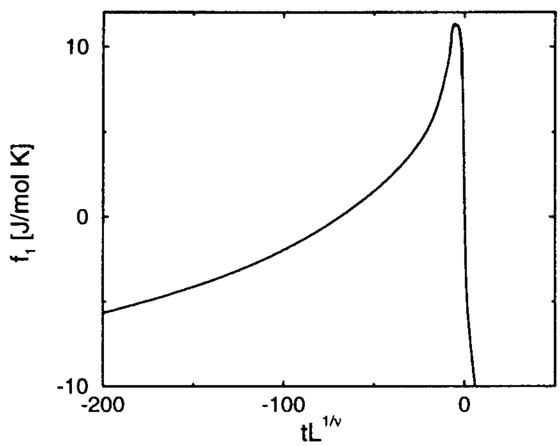

Fig.3: Expected shape of the scaling function f_1 vs $t\, L^{1/\nu}$ with L in Å.

The curves for the surface region above T_λ in Figs. 1(b) and 2(b) are taken from Ref. 3, for cubic geometry and Dirichlet b.c. in one direction. As shown in Ref. 5, $C(t, L)$ depends only weakly on the geometry for $T \geq T_\lambda$. A detailed derivation and a two-loop calculation in half-space geometry with Dirichlet b.c. will be presented elsewhere [8].

The solid lines in Figs. 1-3 in the regime $|x| \lesssim 0$ represent the expected behavior of the specific heat on the basis of an empirical interpolation between the surface parts where we used experimental data [10] in the maximum region in a rescaled form. Additional theoretical work is needed here before quantitative predictions on the specific heat maximum can be made.

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