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Renormalization Group Theory of Critical First Sound along the λ-Line of ^{4}He (*).

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Abstract. – A new model for the critical dynamics of ^{4}He near T_λ including heat and sound modes is introduced. The attenuation and dispersion of the first-sound mode are calculated by means of renormalized field theory. The results provide predictions without adjustable parameters in the entire experimentally accessible frequency and temperature regime and for all pressures along the λ-line. The theory agrees very well with attenuation experiments and explains the discrepancy of an earlier analysis in the high-frequency regime.

The theory of first sound near T_λ in ^{4}He constitutes a complicated problem in the area of critical dynamics which until now has resisted a satisfactory solution. Not even an attempt has been made so far to derive the appropriate equations of motion which should be taken as the starting point for a systematic analysis. Instead FERRELL and BHATTACHARJEE (FB) [1] have used an ingenious phenomenological approach which is based on a frequency-dependent generalization of the specific heat. Although FB have identified the leading mechanism of critical sound attenuation and have obtained results in remarkable agreement with experimental data in an intermediate-frequency range and at low pressures [2] there are obvious reasons why the present status of the theory [1] must be considered as unsatisfactory. First, there exist discrepancies with attenuation data [2-4] at high pressures and in the high- and low-frequency range (see the dashed curves in fig. 2 and 3 below). Second, a statistical foundation is required for the use of a frequency-dependent specific heat. Third, it is desirable to see to what extent the critical dynamics of the first-sound mode can be related to the statics [5] and low-frequency dynamics [6] of ^{4}He described in terms of the well-known model F [6, 7]. This would prepare the ground for a simultaneous treatment of first and second sound below T_λ which is a necessary step towards the resolution of a controversy [8] that is still open in the critical dynamics of ^{4}He.

For these reasons we present a new theory of first sound based on a complete set of Langevin equations which contain model F as a special case. Besides the complex order

(*) Dedicated to Professor RICHARD A. FERRELL on the occasion of his sixtieth birthday.

parameter $\psi_0(x,t)$ and the entropy variable $m_0(x,t)$, we take into account a pressure variable $p_0(x,t)$ and the longitudinal component $j_0(x,t)$ of the momentum density. On the grounds of hydrodynamic equations and renormalization group (RG) arguments we propose the following model:

$$\frac{\partial}{\partial t}\psi_0 = -2\Gamma_0\frac{\delta H}{\delta\psi_0^*} + i\mathring{g}_m\psi_0\frac{\delta H}{\delta m_0} - i\mathring{g}_p\psi_0\frac{\delta H}{\delta p_0} + \theta_\psi\,, \tag{1}$$

$$\frac{\partial}{\partial t}m_0 = \mathring{\lambda}_m\nabla^2\frac{\delta H}{\delta m_0} + L_0\nabla^2\frac{\delta H}{\delta p_0} - 2\mathring{g}_m\,\mathrm{Im}\left(\psi_0^*\frac{\delta H}{\delta\psi_0^*}\right) + \theta_m\,, \tag{2}$$

$$\frac{\partial}{\partial t}p_0 = L_0\nabla^2\frac{\delta H}{\delta m_0} + \mathring{\lambda}_p\nabla^2\frac{\delta H}{\delta p_0} + 2\mathring{G}_p\,\mathrm{Im}\left(\psi_0^*\frac{\delta H}{\delta\psi_0^*}\right) - c_0\nabla\frac{\delta H}{\delta j_0} + \theta_p\,, \tag{3}$$

$$\frac{\partial}{\partial t}j_0 = \mathring{\lambda}_j\nabla^2\frac{\delta H}{\delta j_0} - c_0\nabla\frac{\delta H}{\delta p_0} + \theta_j\,, \tag{4}$$

$$H = \int \mathrm{d}^d x\,[\tfrac{1}{2}(\mathring{\tau}_0|\psi_0|^2 + |\nabla\psi_0|^2 + \mathring{\chi}_m^{-1}m_0^2 + \mathring{\chi}_p^{-1}p_0^2 + \mathring{\chi}_j^{-1}j_0^2) + \\ + \hat{u}_0|\psi_0|^4 + (\mathring{\gamma}_m m_0 + \mathring{\gamma}_p p_0)|\psi_0|^2 - \mathring{h}_m m_0 - \mathring{h}_p p_0]\,. \tag{5}$$

The Langevin forces θ_i have the usual correlations [7]. The parameters $\mathring{g}_m$, $\mathring{\lambda}_m$, Γ_0, $\mathring{\gamma}_m$, and $\mathring{\chi}_m$ keep the same meaning as in model F. A detailed foundation and general discussion of our model will be given elsewhere. Among the various couplings the most important one (as far as first sound is concerned) is $\mathring{\gamma}_p$. It is closely related to $\mathring{\gamma}_m$ according to [9]:

$$\frac{\mathring{\gamma}_p}{\mathring{\gamma}_m} = -k_B^{-1}\bar{\rho}\left(\frac{\partial\bar{\sigma}}{\partial P}\right)_\lambda\,. \tag{6}$$

$\bar{\rho}$ and $\bar{\sigma}$ denote the equilibrium mass density and entropy per unit mass; the derivative is taken along the λ-line. Equation (6) enables us to express the dominant part $\sim\mathring{\gamma}_p^2$ of the sound attenutation in terms of $\mathring{\gamma}_m$ and its renormalized effective counterpart $\gamma_m(l)$ determined previously [5, 10, 11]. Our quantitative knowledge of $\gamma_p(l)$ and $\gamma_m(l)$ is a substantial advantage over the approach of FB[1] in the precritical region.

We wish to calculate the critical dependence of the sound velocity c_1 and damping D_1 on the frequency ω and the relative temperature $t=(T-T_\lambda)/T_\lambda \geq 0$ for small wave numbers k. In this limit $c_1(t,\omega)$ and $D_1(t,\omega)$ can be identified from the denominator $|-\omega^2 + c_1^2k^2 - i\omega D_1 k^2|^{-2}$ of the dynamic structure factor. Instead of D_1, we shall eventually consider

$$\alpha(t,\omega) = \frac{\omega^2}{2c_1(t,\omega)^3}D_1(t,\omega)\,. \tag{7}$$

Keeping only the linear terms of (1)-(4) leads to

$$c_1^{(0)} = c_0/(\mathring{\chi}_j\mathring{\chi}_p)^{\frac{1}{2}}\,, \qquad D_1^{(0)} = \mathring{\lambda}_j/\mathring{\chi}_j + \mathring{\lambda}_p/\mathring{\chi}_p\,. \tag{8}$$

In order to treat the nonlinear interactions, we employ a field-theoretic approach using response fields $\tilde{\psi}_0$, $\tilde{m}_0$, $\tilde{p}_0$, $\tilde{j}_0$ [12, 13]. In this approach c_1 and D_1 can be expressed in terms of two-point vertex functions $\mathring{\Gamma}_{\varphi\tilde{\varphi}}(k, t, \omega)$ according to

$$c_1(t, \omega)^2 = \mathrm{Re}\,[y(t, \omega) - i\omega z(t, \omega)]\ , \tag{9}$$

$$D_1(t, \omega) = \frac{1}{\omega}\,\mathrm{Im}\,[i\omega z(t, \omega) - y(t, \omega)]\ , \tag{10}$$

$$y(t, \omega) = -\frac{\partial}{\partial k^2}(\mathring{\Gamma}_{j\tilde{p}}\mathring{\Gamma}_{p\tilde{j}})|_{k=0}\ , \tag{11}$$

$$z(t, \omega) = \frac{\partial}{\partial k^2}(\mathring{\Gamma}_{j\tilde{j}} + \mathring{\Gamma}_{p\tilde{p}} + \mathring{\Gamma}_{m\tilde{j}}\mathring{\Gamma}_{p\tilde{m}}\mathring{\Gamma}_{p\tilde{j}}^{-1})|_{k=0}\ . \tag{12}$$

The vertex functions $\mathring{\Gamma}_{\varphi\tilde{\varphi}}$ are conveniently treated by means of the minimal renormalization [13]. Due to the conservation property of the fields m_0, p_0, j_0, the unrenormalized vertex functions on the r.h.s. of (11) and (12) can be replaced directly by the renormalized ones $\Gamma_{\varphi\tilde{\varphi}}$. The final step is to integrate the RG equation. This amounts to replacing the renormalized parameters by effective parameters which satisfy RG flow equations.

By inspection of the diagrams, it is seen that the dynamic couplings and γ_m enter $y(t, \omega)$ only in two-loop order. Therefore, (11) can be rewritten as

$$y(t, \omega) = c(l)^2[1 - \gamma_p(l)^2\Sigma(t, \omega)]\ , \tag{13}$$

where $c(l)$ is the effective renormalized counterpart of c_0. In (12) we shall neglect the small contributions related to g_p and G_p. Then we obtain

$$z(t, \omega) = \lambda_j + \lambda_p[1 - \gamma_p^2\Sigma] - 2L\gamma_p\gamma_m\Sigma + \lambda_m\frac{(\gamma_p\gamma_m\Sigma)^2}{1 - \gamma_p^2\Sigma} \tag{14}$$

with $\gamma_i \equiv \gamma_i(l)$, $\lambda_i \equiv \lambda_i(l)$, $L \equiv L(l)$ and the same function $\Sigma(t, \omega)$ as in (13). The remaining task is to determine $\Sigma(t, \omega)$ and the dependence of various effective parameters on the flow parameter $l = l(t, \omega)$.

We have derived general relations between $\gamma_m(l)$, $\gamma_p(l)$ and the specific heat C analogous to those obtained recently [5]. Using (6) we are able to express $\gamma_p(l)$ entirely in terms of C and the (experimentally known) background susceptibilities $\mathring{\chi}_m$ and $\mathring{\chi}_p$. The resulting $\gamma_p(l)^2$ is shown in fig. 1. We emphasize that our $\gamma_p(l)^2$ remains quantitatively reliable well outside the asymptotic regime. Since $\gamma_p(l) \ll \gamma_m(l)$, the present $\gamma_m(l)$ is almost identical with the previous [5, 6, 10] $\gamma_m(l)$ of model F.

Finally we turn to $\Sigma(t, \omega)$. Invoking a fluctuation-dissipation theorem [6], we are led to separate purely dissipative contributions from genuine dynamic ones. This is achieved by rewriting Σ, without loss of generality, as

$$\Sigma(t, \omega) = \frac{1 + P(t, \omega)}{\gamma_m(l)^2 + \gamma_p(l)^2}\left[1 - \frac{1}{1 + [\gamma_m(l)^2 + \gamma_p(l)^2]\,G_+(t, \omega)}\right] \tag{15}$$

with a dynamic part $P(t, \omega)$ and a purely dissipative part $G_+(t, \omega)$ which depends only on γ_m, γ_p, u (compare the analogous case of eq. (4.5) of ref. [6]. The latter can be identified, at $\omega = 0$, as $G_+(t, 0) = F_+[u(l), 1]$, where F_+ characterizes the u-dependence of the specific heat above T_λ [5]. In one-loop order $P(t, \omega)$ vanishes. In this order we have calculated G_+ (in lowest order of an $\varepsilon = 4\text{-}d$ expansion) as

$$G_+(t,\omega) = -\frac{2}{\Omega(l)}\operatorname{arctg}\Omega(l) + i\left[2\operatorname{arctg}\Omega(l) - \frac{\ln(1+\Omega(l)^2)}{\Omega(l)}\right], \tag{16}$$

where $\Omega(l) = \omega/2\Gamma'(l)r(l)$ with $l = l(t,\omega)$ determined by

$$\left|\frac{r(l)}{\mu^2 l^2} - \frac{i\omega}{2\Gamma'(l)\mu^2 l^2}\right| = 1\,. \tag{17}$$

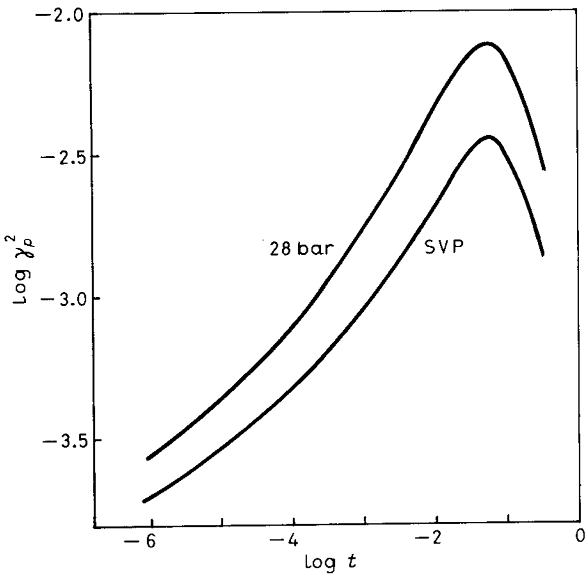

Fig. 1. – Effective coupling $\gamma_p = \gamma_p(l(t,0))$ *vs.* reduced temperature $t = (T - T_\lambda(P))/T_\lambda(P)$ for SVP and 28 bar.

Here $r(l)$ is known from statics [5, 10, 11]. Furthermore, the present $\Gamma'(l)$ is well represented by the previous $\Gamma'(l)$ of model F. The reason is that the dependence of R_λ^{eff} on $w'(l) = \Gamma'(l)/\lambda_m(l)$ remains essentially the same within the present model.

This completes our calculation of $\alpha(t,\omega)$, up to a background contribution $\alpha^{(0)}$ determined by (8). For the comparison with experiment, we shall need only the critical part $\alpha^c = \alpha - \alpha^{(0)}$.

Since the contribution of (14) (which is not contained in ref. [1]) turns out to be almost negligible, it suffices, for the present purpose, to represent α^c by the simplified expression arising from (13), (15)-(17)

$$\alpha^c(t,\omega) = \frac{\omega}{2c_1(t,\omega)}\,\gamma_p(l)^2\,\mathrm{Im}\left[\frac{G_+(t,\omega)}{1 + [\gamma_m(l)^2 + \gamma_p(l)^2]\,G_+(t,\omega)}\right] \tag{18}$$

apart from negligible corrections of $O(\gamma_p^4)$. This result is plotted as a function of t in fig. 2 and 3 (solid curves) for 1 MHz and 1 GHz, where reliable experimental data [2-4] are available. The agreement of our theory with the data is excellent.

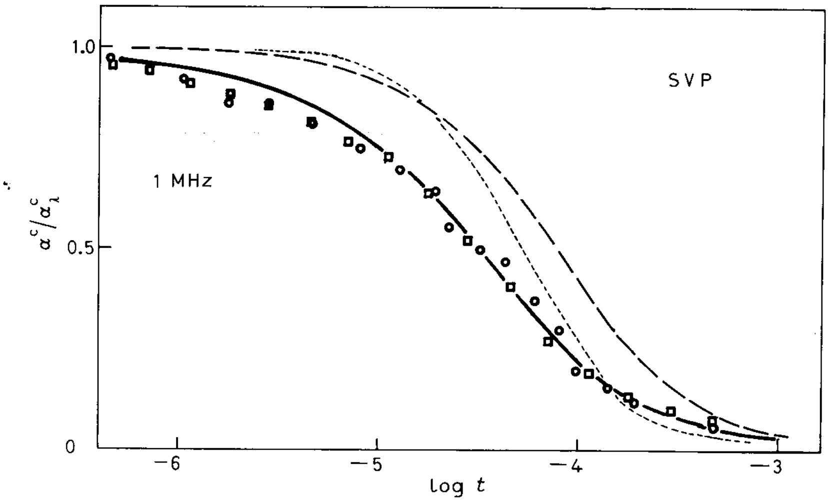

Fig. 2. – Critical part of the attenuation α^c, normalized to its value α^c_λ at the λ-point, *vs.* t at SVP and for $\omega/2\pi = 1$ MHz. The data are taken from ref. [3] (□), ref. [4] (○). The solid curve is our eq. (18); the dashed curve from eq. (13) of ref. [1]; thin dotted curve is an earlier theoretical prediction represented by the dashed curve of fig. 16 of ref. [3].

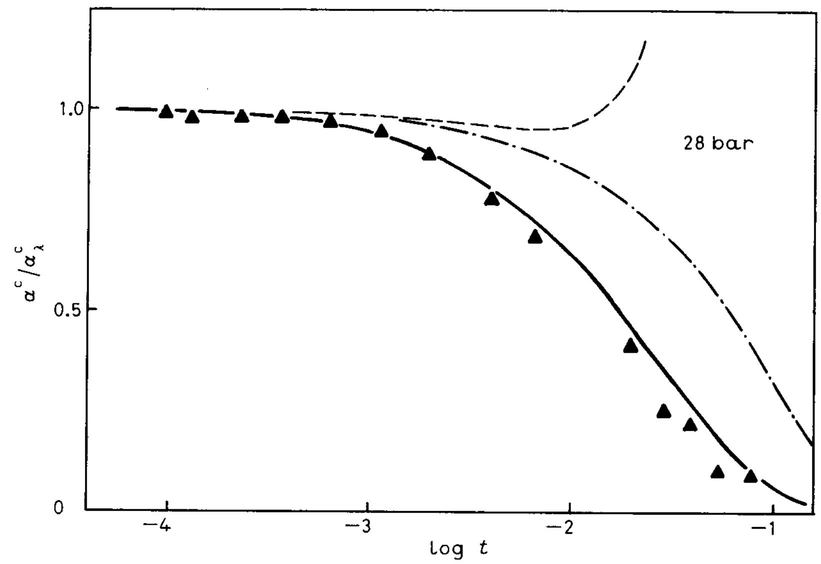

Fig. 3. – Critical part of the attenuation, normalized to its value at $T_\lambda(P)$, *vs.* t at 28 bar and for $\omega/2\pi = 1$ GHz. The data are the full triangles of fig. 1*a*) or fig. 2*b*) of ref. [2]. Solid curve is our eq. (18). Using eq. (13) of ref. [1] yields the dashed curve which is also represented by the full curve in fig. 2*b*) of ref. [2]. The dot-dashed curve is our eq. (18) for $\omega/2\pi = 10$ GHz and 28 bar.

An application of the theory of FB in its present form (eq. (13) of ref. [1]) yields the dashed curve in fig. 2. On the basis of our theory the disagreement with the data is explained by the appreciable l-dependence of $\Gamma''(l)$ in the experimental range which has been neglected by FB[1]. This is justified only in the range $\omega/2\pi \geqslant 10$ MHz (rather than 1 MHz claimed by FB) where the FB results are essentially confirmed by our theory.

In the high-frequency regime, the FB results have been employed in a data analysis by VIDAL *et al.* [2]. They observed a significant discrepancy between the data at 1 GHz and eq. (13) of FB[1] (solid curve in fig. 2*b*) of ref. [2], dashed curve in our fig. 3) ([1]).

We attribute this disagreement to the breakdown of the logarithmic representation of the specific heat [1] rather than to a fundamental failure of the FB approach itself.

In closing, we briefly report good agreement of our theory also with the frequency dependence of the attenuation data [2-4] at $T_\lambda(p)$. On the basis of eq. (18) we predict that $\alpha^c \cdot c_1/\omega$ reaches a maximum value of about 0.01 at a frequency of roughly 8 GHz (for 28 bar) which corresponds to the maximum of $\gamma_p(l)^2$ in fig. 1. In fig. 3 the temperature dependence of the attenuation at 10 GHz (dot-dashed curve) is predicted. An experimental verification of these high-frequency crossover effects would constitute a novel test of the theory. A more detailed analysis including the low-frequency regime and the velocity dispersion will be given elsewhere.

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([1]) We note that this discrepancy is not adequately exhibited in fig. 2*b*) of ref. [2], since the data have been multiplied by a «nonscaling factor».

REFERENCES

[1] FERRELL R. A. and BHATTACHARJEE J. K., *Phys. Rev. Lett.*, **44** (1980) 403; see also *Phys. Rev. B*, **25** (1982) 3168.
[2] VIDAL F., TARVIN J. A. and GREYTAK T. J., *Phys. Rev. B*, **25** (1982) 7040.
[3] CAREY R., BUCHAL CH. and POBELL F., *Phys. Rev. B*, **16** (1977) 3133.
[4] WILLIAMS D. and RUDNICK I., *Phys. Rev. Lett.*, **25** (1970) 276.
[5] DOHM V., *Phys. Rev. Lett.*, **53** (1984) 1379; *Z. Phys. B*, **60** (1985) 61.
[6] DOHM V., *Z. Phys. B*, **61**, 193 (1985).
[7] HALPERIN B. I., HOHENBERG P. C. and SIGGIA E. D., *Phys. Rev. B*, **13** (1976) 1299.
[8] AHLERS G. and HOHENBERG P. C., *Phys. Rev. Lett.*, **52** (1984) 313; FERRELL R. A. and BHATTARCHARJEE J. K., *Phys. Rev. Lett.*, **52** (1984) 314.
[9] ONUKI A., *J. Low Temp. Phys.*, **53** (1983) 1.
[10] TAM W. Y. and AHLERS G., *Phys. Rev. B*, **33** (1986) 183.
[11] SCHLOMS R. and DOHM V., manuscript in preparation.
[12] JANSSEN H. K., *Z. Phys. B*, **23** (1976) 377; BAUSCH R., JANSSEN H. K. and WAGNER H., *Z. Phys. B*, **24** (1976) 113.
[13] DE DOMINICIS C. and PELITI L., *Phys. Rev. B*, **18** (1978) 353.