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Heat flow induced anomalies of the entropy and specific heat near T_λ of ^{4}He

Rudolf Haussmann[a] and Volker Dohm[b]

[a] Department of Physics, Indiana University, Bloomington, Indiana 47405, USA
[b] Institut für Theoretische Physik, Technische Hochschule Aachen, D-52056 Aachen, Germany

The effect of a stationary heat current Q on superfluid ^{4}He in a homogeneous metastable state near $T_\lambda(Q)$ is studied. On the basis of a renormalization-group calculation of the entropy we predict a considerable enhancement $\Delta C_{J_s} = C_{J_s}(T,Q) - C_{J_s}(T,0)$ of the specific heat C_{J_s} at constant superfluid current J_s up to a critical heat current $Q_c(T)$ where C_{J_s} is divergent. This result is compared with the cusp anomaly of ΔC_{v_s} of the specific heat C_{v_s} at constant superfluid velocity. In the limit $Q \to 0$ we predict the universal ratio $\lim_{Q\to 0} \Delta C_{J_s}/\Delta C_{v_s} = (1+\nu)/(1-\nu) = 5.08$.

1. Introduction

In this paper we study superfluid ^{4}He in a homogeneous metastable state near $T_\lambda(Q)$ [1] in the presence of a stationary heat current $\mathbf{Q} = -(\rho_s/\rho)ST\mathbf{w}$ where $\mathbf{w} = \mathbf{v}_s - \mathbf{v}_n$ is the velocity of the superfluid-normal fluid counterflow, ρ_s/ρ is the superfluid fraction and S is the entropy per unit volume. Recently [2] we have performed a renormalization group (RG) calculation of the specific heat $C_{v_s}(T,Q)$ at constant $\mathbf{w} = \mathbf{v}_s$ (in the frame where $\mathbf{v}_n = 0$) as a function of Q up to the critical heat current [1] $Q_c(T)$ where C_{v_s} was predicted to exhibit a cusp-like anomaly. Stimulated by Ref. 3, we have extended this RG study to the specific heat $C_{J_s}(T,Q)$ at constant superfluid current $J_s = \rho_s v_s$ and have found a divergence of C_{J_s} as $Q_c(T)$ is approached [4]. Subsequently, qualitatively similar results were found in Ref. 5. Here we shall present the derivation and a further discussion of the results of Ref. 4.

2. Entropy and specific heat

The entropy S near $T_\lambda(Q)$ can be described on the basis of the static probability distribution of models C or F [6] according to

$$S = \langle m \rangle = \chi_0 h_0 - \chi_0 \gamma_0 \langle |\psi|^2 \rangle \tag{1}$$

where ψ is the complex order parameter. The (bare) average $\langle |\psi|^2 \rangle$ has been calculated previously [2] in a perturbation expansion around the plane-wave order parameter $\langle \psi \rangle = \eta \exp(i\mathbf{kx})$ at finite superfluid velocity $\mathbf{v}_s = \hbar\mathbf{k}/m$. Application of the RG theory in d dimensions [6, 7] is parallel to the previous treatment [1, 2] and yields, for $\kappa \le \kappa_c \simeq 0.4$,

$$S(t,\kappa) = S_\lambda + t\{B + \bar{A}(1-\alpha)^{-1}E_-(\kappa)(-t)^{-\alpha}\} \tag{2}$$

with the amplitude function

$$E_-(\kappa) = \frac{4\nu}{\alpha} + \frac{1-2\kappa^2}{2u^*} - \frac{8}{\epsilon}(1-3\kappa^2) + \frac{8}{\epsilon}(1-2\kappa^2)^{(d-2)/2} F\left(\frac{2-d}{2}, \frac{1}{2}; \frac{d}{2}; y^2\right) \tag{3}$$

where $\epsilon = 4-d$, $t = (T-T_\lambda)/T_\lambda$, $\kappa = k\xi(-2t)$, $y = 2\kappa/(1-2\kappa^2)^{1/2}$, and $F(a,b;c;z)$ is the hypergeometric function. For $d = 3$ the result is

$$E_-(\kappa) = 4\nu/\alpha + (1-2\kappa^2)/2u^* - 8(1-3\kappa^2) + 4(1-2\kappa^2)^{1/2}[(1-y^2)^{1/2} + y^{-1}\arcsin(y)] \tag{4}$$

where $u^* = 0.0362$ and $\xi(t) = \xi_0 t^{-\nu}$ is the correlation length above T_λ with $\xi_0 = 1.4$ Å. For $\kappa = 0$, $C = dS/dt$ yields the equilibrium specific heat with the parameters $\alpha = -0.01285, \nu = (2-\alpha)/3, B = 456.3 J/mol\ K$ and $\bar{A} = 2.275 J/mol\ K$ taken from [8]. For $\kappa \ne 0$ we obtain C_{J_s} from (2) - (4) as

$$C_{J_s}[t,\kappa] = dS(t,\kappa[t,J_s])/dt \tag{5}$$

$$= \partial S(t,\kappa)/\partial t + \{\partial S(t,\kappa)/\partial\kappa\}\partial\kappa[t,J_s]/\partial t \tag{6}$$

$$= B + \bar{A}E_-(\kappa)(-t)^{-\alpha}\left(1 - \frac{2\nu[\ell n\, E_-(\kappa)]'}{(1-\alpha)[\ell n\, f_J(\kappa)]'}\right). \tag{7}$$

The prime denotes differentiation with respect to κ. The function $\kappa[t,J_s]$ is defined by inverting $J_s(t,\kappa) = \xi(-2t)^{-2} f_J(\kappa)$ where the scaling function $f_J(\kappa)$ is known in one-loop order [1]. Finally we substitute $\kappa = \kappa(T,Q)$, as determined previously [2], to arrive at $C_{J_s}(T,Q) = C_{J_s}[t,\kappa(T,Q)]$. Similarly $C_{v_s}(T,Q)$ is obtained by using $\kappa(t) = (m/\hbar)v_s\xi_0(-2t)^{-\nu}$ at constant v_s instead of $\kappa[t,J_s]$. The enhancement $\Delta C_i(T,Q) = C_i(T,Q) - C(T,0)$ is universally described by the scaling functions [2] $f_i(Q/Q_c) = (-t)^\alpha \Delta C_i(T,Q)$. In Fig. 1 we show our results for $f_{v_s}(Q/Q_c)$ [2] and $f_{J_s}(Q/Q_c)$ [4].

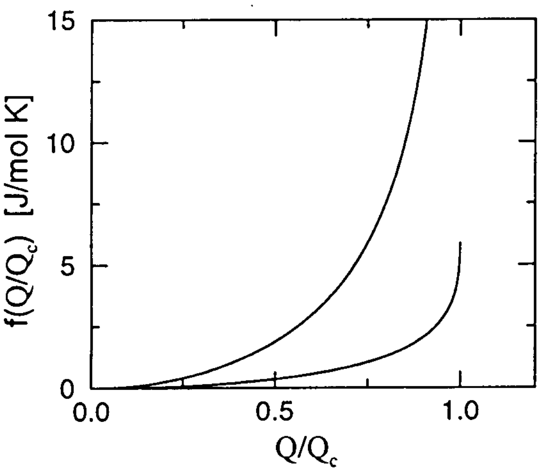

Figure 1: Scaling functions of the specific-heat enhancement $f(Q/Q_c) = (-t)^\alpha \Delta C(T,Q)$ for constant v_s (lower line) and constant J_s (upper line).

3. Discussion

For small $Q \ll Q_c$ the scaling functions can be expanded as $f_i(Q/Q_c) = a_i\,(Q/Q_c)^2 + \cdots$ where the amplitudes a_i have the (exact) universal ratio

$$a_{J_s}/a_{v_s} = (1+\nu)/(1-\nu). \tag{8}$$

Thus we predict $a_{J_s}/a_{v_s} \approx 5.08$. Within our approximation we obtain $a_{v_s} = 1.23 J/mol\,K$ and $a_{J_s} = 6.25 J/mol\,K$.

Since E_-, E'_- and f_J remain finite for $\kappa \to \kappa_c$ corresponding to $Q \to Q_c$, the divergence of C_{J_s} in (7) is governed by $[f'_J(\kappa)]^{-1}$ near κ_c where f'_J vanishes [1]. As indicated by the one-loop analysis [1] the singularity appears at $\kappa_c < \kappa_c^{mf} = 6^{-1/2}$ and is presumably not mean-field like. The line $Q_c(T)$ or $T_\lambda(Q)$ constitutes a borderline of ultimate (meta)stability of the homogeneous superfluid phase. Since ρ_s and ξ_T^- remain finite at this line [1] we consider as incorrect the interpretation [5] of $T_\lambda(Q)$ as a line of critical points.

The question of an experimental verification of our predictions has not yet been investigated previously [2-5]. Since Q is controllable quite accurately, a *thermodynamic* measurement of $C_Q = C_{J_s}(T,Q)$ at constant $Q = -k_B T_\lambda g_0 J_s$ is presumably easier than that of C_{v_s}. As an alternative, C_{v_s} may be measurable in a *dynamic* (second-sound) experiment [4]. If the (in-phase) oscillations ΔT and ΔQ of the temperature and heat current can be measured together with the second-sound velocity c_2 then the specific heat is obtained as

$$C = c_2^{-1}\Delta Q/\Delta T\,. \tag{9}$$

As shown previously [9], $c_2 = c_2(\theta)$ depends on the angle θ between the second-sound propagation and the heat current. Thus, $C = C(\theta)$ obtained by (9) will also depend on this angle. By inspection of the linearized hydrodynamic equations in Ref. 9 we find that v_s is constant (up to linear order) if $\theta = \pm\pi/2$. Thus, an experiment with $\mathbf{c}_2$ perpendicular to $\mathbf{Q}$ might be a possibility to measure $C_{v_s}(T,Q)$.

In all theoretical considerations [1-5, 9] the metastability of the superfluid quasi-equilibrium state at finite Q has been neglected. This metastability may cause dissipation due to fluctuations. Thus the predicted specific-heat anomalies ΔC near $T_\lambda(Q)$ may be partly masked by the onset of dissipation at $T_c(Q) < T_\lambda(Q)$ as detected recently [10].

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