DYNAMIC SCALING AND CORRECTIONS TO SCALING NEAR BICRITICAL AND TETRACRITICAL POINTS

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ABSTRACT

We study the critical dynamics of stochastic models appropriate for describing bicritical and tetracritical points in anisotropic antiferromagnetic systems. The dynamic exponents and the transient exponents are calculated by renormalized field theory up to two-loop order. In the presence of reversible mode-coupling terms, two-loop contributions establish bicritical dynamic scaling in the restricted sense and invalidate recent predictions based on mode-coupling arguments. In the case of an n-component relaxational model total dynamic scaling is found to $O(\varepsilon^2)$ both at the bicritical ($n \leq 3$) and the tetracritical ($4 \leq n < 11$) points.

INTRODUCTION

Multicritical phenomena, in particular near bicritical points[1], have received increasing attention in recent years. An n-component antiferromagnetic model exhibiting bicritical ($n \leq 3$) and tetracritical ($4 \leq n < 11$) behavior was analyzed by means of the renormalization-group ε expansion approach[1], and static bicritical and tetracritical scaling was found to be valid to $O(\varepsilon)$ for $n<11$. In this work we investigate the validity of the dynamic scaling hypothesis[2] near bicritical and tetracritical points and report the first renormalization group analysis of this problem. This is of particular interest in view of a recent prediction [3] given on the basis of mode-coupling and scaling arguments for the n=3 spin-flop bicritical point in uniaxial antiferromagnets having rotational symmetry around the anisotropy axis. While this prediction implies a breakdown of dynamic scaling, we find (restricted) dynamic scaling to be valid due to the appearance of nonanalytic two-loop contributions similar to those in model C of Halperin, Hohenberg and Ma (HHM)[4].

We formulate an n-component dynamic model in terms of the Lagrangian version of the Martin-Siggia-Rose formalism[5], as given previously by one of us[6], and employ the field-theoretic renormalization-group approach of Bausch, Janssen, and Wagner[7], combined with the minimal subtraction procedure[8]. In the presence of reversible mode-coupling terms we find that bicritical dynamics must be treated at least up to two-loop order in order to avoid a spurious breakdown of scaling at $O(\varepsilon=4-d)$ and to work out the leading deviations from conventional theory. Our analysis will show that bicritical dynamic scaling, at least in the restricted sense, as well as tetracritical dynamic scaling are indeed valid in all cases as long as static scaling holds ($n<11$). Furthermore our results provide additional motivation for reexamining tricritical dynamics in He^3-He^4 mixtures[9] by a two-loop analysis.

BICRITICAL AND TETRACRITICAL MODELS

We start from the Hamiltonian

$$H = \frac{1}{2}\int d^d x\,[r_\parallel \vec{\sigma}^2 + (\vec{\nabla}\vec{\sigma})^2 + r_\perp \vec{s}^2 + (\vec{\nabla}\vec{s})^2 + m^2 + h\,m + U(\vec{\sigma}^2)^2 + 2W\,\vec{\sigma}^2\vec{s}^2 + V(\vec{s}^2)^2 + A\,m\,\vec{\sigma}^2 + B\,m\,\vec{s}^2] \quad (1)$$

which is identical with that of Kosterlitz et al.[1] except for the additional terms containing the one-component nonordering density m. In a dynamic theory these terms must be treated explicitly provided that m is a conserved density (e.g., the z-component of the magnetization of an $n_\parallel + n_\perp = 3$ - component uniaxial antiferromagnet in an external homogeneous magnetic field h). $\vec{\sigma}(\vec{x})$ and $\vec{s}(\vec{x})$ denote $n_\parallel$ -component and $n_\perp$-component order-parameter densities, respectively. Our dynamic model is defined by the action integral[6] $J = \int dt\, L$ with the Lagrangian[10]

$$L = \int d^d x\, [\tilde{\sigma}_\alpha (L_\sigma \tilde{\sigma}_\alpha - \dot{\sigma}_\alpha - L_\sigma\, \delta H/\delta\sigma_\alpha) + \tilde{s}_\alpha(L_s \tilde{s}_\alpha - \dot{s}_\alpha - L_{\alpha\beta}\,\delta H/\delta s_\beta + G_{\alpha\beta}\, s_\beta\, \delta H/\delta m) + \tilde{m}(-L_m \vec{\nabla}^2\tilde{m} - \dot{m} + L_m \vec{\nabla}^2\,\delta H/\delta m - G_{\alpha\beta}\, s_\beta\, \delta H/\delta s_\alpha)] \quad (2)$$

(summation over repeated indices is implied). The variables $\tilde{\vec{\sigma}}(\vec{x}t)$, $\tilde{\vec{s}}(\vec{x}t)$, and $\tilde{m}(\vec{x}t)$ are response fields. The L_i's are kinetic coefficients, $L_{\alpha\beta} = L_s\,\delta_{\alpha\beta} + F_{\alpha\beta}$; $G_{\alpha\beta}$ and $F_{\alpha\beta}$ are antisymmetric mode-coupling matrices. All relevant couplings are retained in (2) as can be seen from dimensional arguments applied to J, in complete analogy to the static case.

We shall examine the following models: (I) $n_\parallel = 1$, $n_\perp = 2$, with $G_{12} = G$, $F_{12} = F$; (II) $n_\parallel$ and $n_\perp$ arbitrary but m not conserved. Applications are the spin-flop bicritical points in MnF_2[11] (model I) and in $GdAlO_3$[12] or in $CuCl_2 \cdot 2H_2O$[13] (model II with $n_\parallel = n_\perp = 1$) and possibly tretracritical points in higher-component systems (model II with $n_\parallel + n_\perp \geq 4$).

RENORMALIZED FIELD THEORY

As outlined in a recent letter [14], we proceed according to renormalized field theory [7] by introducing renormalized fields $\phi_j = Z_j^{-1/2} j$ ($j = \vec{\sigma}, \vec{s}$, m, $\tilde{\vec{\sigma}}, \tilde{\vec{s}}, \tilde{m}$) and a set p_i of renormalized dimensionless parameters. We need in particular the ratios $\rho_\sigma = \lambda_\sigma/\lambda_m$ and $\rho_s = \lambda_s/\lambda_m$ with $\lambda_i = Z_{\lambda_i} L_i$, and the mode-coupling constants $g = G/(4\pi\, \mu^{\varepsilon/2}\lambda_m Z_g)$, $f = F/(4\pi\,\mu^{\varepsilon/2}\lambda_m Z_f)$; here μ^{-1} is the usual parameter defining a length scale in the renormalized theory. In determining the Z-factors we have taken advantage of the minimal renormalization procedure[8] which has several advantages[15,14].

RESULTS: MODEL I

While the one-loop approximation yields the same dynamic exponents as predicted by Hubèr and Raghavan[3], we have found that the two-loop contributions cannot be neglected but lead to a new stable fixed point for arbitrarily small ε. This is due to the appearance of logarithmic terms similar to those in model C of HHM[4]. At two loop order, the dynamic fixed point values are (for statics see [1] and [14])

$$\rho_s^* = \rho_\sigma^* = 0\ ,\ g^{*2}/\rho_s^* = 9\varepsilon/11\ , \quad (3)$$

with the finite ratio $\rho_\sigma^*/\rho_s^* \sim \exp(-198/\varepsilon)$. This implies identical dynamic exponents z_σ and z_s for the "parallel" and "perpendicular" order-parameter correlation function and restores bicritical dynamic scaling in the restricted sense. Extended scaling is violated due to the different dynamic exponent z_m govern-

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ing the m-m correlation function. We find to two-loop order

$$z_\sigma = z_s = 2 + 0.0007\varepsilon^2 + O(\rho_\sigma^*/\rho_s^*) \quad , \tag{4}$$

and from a dissipation-fluctuation theorem

$$z_m = \frac{\phi}{\nu} \tag{5}$$

which is an exact result. With respect to the variables ρ_s , ρ_σ , $x \equiv g^2/\rho_s$ and $y \equiv f/\rho_s^{1/2}$ the transient exponents (governing the corrections to scaling) are to $O(\varepsilon)$

$$\omega_{\rho_s} = \omega_{\rho_\sigma} = 2\varepsilon/11 \text{ , } \omega_x = 7\varepsilon/11 \text{ , } \omega_y = 19\varepsilon/11 \quad . \tag{6}$$

RESULTS: MODEL II

Since m is assumed to be a nonconserved density, the critical dynamics of the order parameter is not affected by the m-variable. Therefore we drop the corresponding terms in (1) and (2). Now the only dynamic parameter of interest is the ratio $\lambda \equiv \lambda_\sigma/\lambda_s$. It has different fixed point values λ_H^* or λ_B^* depending on whether static Heisenberg or biconical fixed point values[1] are used.

For the Heisenberg fixed point we obtain $\lambda_H^* = 1$, independent of $n = n_\parallel + n_\perp$. The corresponding dynamic exponent Z_H and transient exponent ω_H are

$$Z_H = 2 + 0.7261\ \eta \text{ , } \omega_H = \frac{2\ n\ \varepsilon^2}{(n+8)^2}\ \ell n\ \frac{4}{3} \tag{7}$$

with $\eta = \frac{1}{2}(n+2)\ \varepsilon^2/(n+8)^2 + O(\varepsilon^3)$.

For the biconical fixed point[1] we consider $n_\parallel=1$, $n_\perp = n-1$. The corresponding values $\lambda_B^*(n)$, $z_B^*(n)$, and $\omega_B^*(n)$ are given in Table I, as well as the Heisenberg exponents(7) (compare Table I of Kosterlitz et al.[1]).

TABLE I

Dynamic biconical and Heisenberg exponents for model II evaluated to order ε^2 at ε=1

	Biconical			Heisenberg	
n	$\lambda_B^*(n)$	$z_B(n)$	$\omega_B(n)$	$z_H(n)$	$\omega_H(n)$
1	1.1145	2.0135	0.0126	2.0135	0.0071
2	1.0000	2.0135	0.0320	2.0145	0.0115
3	1.0241	2.0149	0.0229	2.0150	0.0143
4	1.0000	2.0151	0.0160	2.0151	0.0160
5	1.0205	2.0151	0.0105	2.0150	0.0170
6	1.0846	2.0149	0.0065	2.0148	0.0176
7	1.2131	2.0147	0.0038	2.0145	0.0179
9	2.1588	2.0141	0.0009	2.0138	0.0179
10	6.0005	2.0138	0.0002	2.0135	0.0178
11	...	2.0135	0	2.0131	0.0175
13	0.0090	2.0129	0.0001	2.0124	0.0170
15	0.1271	2.0122	0.0002	2.0117	0.0163

CONCLUSIONS

We have shown that up to two-loop order bicritical and tetracritical dynamic scaling, respectively, are valid both for model I (in the restricted sense) and for model II (as long as static scaling holds). Thus in the asymptotic scaling region there exist characteristic frequencies for the "parallel" and "perpendicular" order-parameter correlation function governed by a common dynamic exponent z and common transient exponents ω_i. It would be interesting to test this prediction experimentally by NMR-measurements and inelastic neutron scattering near the spin-flop bicritical points in MnF_2 ($z = 2.00$, $z_m = \phi/\nu = 1.78$) and $GdAlO_3$ or $CuCl_2 \cdot 2H_2O$ ($z = 2.015$).

Besides model C of HHM[4] - and perhaps the tricritical model of Siggia and Nelson [9] - our bicritical model I represents a novel example where a nonanalytic ε-dependence appears and two-loop contributions cause a qualitative change of $O(\varepsilon)$-results for arbitrarily small ε. In this context it should be pointed out that, similar to β_ρ of [14] the two-loop β-function of model C as given by De Dominicis et al.[4] allows for a stable fixed point also for $2<n<4$, with a finite value $\lambda^*\sim\exp(-c/\varepsilon)$, $c>0$, in addition to the (unstable) fixed point $\lambda^*=0$. One of the differences between model I and model C is that our $O(\varepsilon)$-fixed point is not only destablized at two-loop order but is no longer a fixed point at this order. Although we have some confidence in the relevance of our two-loop results for model I at least in a qualitative sense, we cannot rule out the possibility that higher-loop terms even restore extended bicritical scaling and also drive the numerical value of ρ_σ^*/ρ_s^* to a less fantastic order of magnitude.

Finally we briefly touch on the question as to the experimental accessibility of the asymptotic dynamic scaling region. Although our theory cannot answer this question quantitatively we suspect that, according to the extremely small ratio ρ_σ^*/ρ_s^*, an n=3 system like MnF_2 may show an effective behavior corresponding to the $O(\varepsilon)$ fixed point ($\rho_\sigma^*/\rho_s^*=0$), i.e. with two different effective exponents z_σ=2.00 and $z_s=z_m=\phi/\nu$=1.78, even very close to the bicritical point. We hope that future experiments in systems like MnF_2 are sufficiently accurate to answer this question and eventually show an ultimate crossover to the asymptotic dynamic exponent(4).

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