PROCEEDINGS

OF THE

JOINT Xth EUROPEAN AND VIth RUSSIAN SYMPOSIUM

ON

PHYSICAL SCIENCES IN MICROGRAVITY

St. PETERSBURG, RUSSIA, 15-21 June 1997

EDS. V.S. AVDUYEVSKY, V.I. POLEZHAEV

VOLUME I

FLUID PHYSICS

FUNDAMENTAL PHYSICS

THERMOPHYSICS AND COMBUSTION

MOSCOW, RUSSIA
1997

CRITICAL-POINT UNIVERSALITY TEST ALONG THE LAMBDA LINE OF ^{4}HE *

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The superfluid transition of ^{4}He at the λ-line $T_\lambda(P)$ is ideal for testing the universality predictions of the renormalization-group (RG) theory. High-precision measurements of the superfluid density ρ_s and of the specific heat C in a microgravity environment and at reduced gravity are planned for future research [1]. The quantities of interest are not only the critical exponents ν and α but also universal ratios of the amplitudes A_ρ, $A^\pm$, a_ρ, $a_c^\pm$ appearing in the asymptotic representations

$$\rho_s = k_B T_\lambda (m/\hbar)^2 A_\rho |t|^\nu \left(1 + a_\rho |t|^\Delta + \cdots\right), \tag{1}$$

$$C^\pm = B + \frac{A^\pm}{\alpha} |t|^{-\alpha} \left(1 + a_c^\pm |t|^\Delta + \cdots\right), \tag{2}$$

where $t = (T - T_\lambda)/T_\lambda$. Accurate calculations of these ratios are needed in order to minimize the number of fitting parameters in a combined analysis of ρ_s and C. For ρ_s such calculations have previously not been carried out beyond one-loop order because of the complications due to Goldstone singularities.

In an effort to improve the theoretical prediction on these amplitude ratios we have recently performed a two-loop RG calculation of ρ_s [2]. Our approach is based on the statistical distribution $\sim \exp(-\mathcal{H})$ with the Landau-Ginzburg-Wilson functional

$$\mathcal{H} = \int \mathrm{d}^d x \left(\frac{1}{2} r_0 |\psi|^2 + \frac{1}{2} |\nabla\psi|^2 + u_0 |\psi|^4\right) \tag{3}$$

for the complex order parameter $\psi(\mathbf{x})$ (Bose condensate wave function). The superfluid density can be calculated via

$$\rho_s = k_B T_\lambda (m/\hbar)^2 |\langle\psi\rangle|^2 \frac{\partial}{\partial k^2} \chi_T(k)^{-1}\Big|_{k=0} \tag{4}$$

*Proceedings of the Joint X^{th} European and VI^{th} Russian Symposium on Physical Sciences in Microgravity, St. Petersburg, 15-21 June 1997.

where $\chi_T(k)$ is the transverse susceptibility at finite wave number k. The result of our calculation in $d = 3$ dimensions can be represented as

$$\rho_s = k_B T_\lambda (m/\hbar)^2 \xi^{-1} f_\psi(u) f_T(u) \tag{5}$$

where

$$\xi_- = \xi_0^- |t|^{-\nu} \left(1 + a_\xi^- |t|^\Delta + \cdots\right) \tag{6}$$

is an appropriately defined correlation length below T_λ [3]. The amplitude functions $f_\psi(u)$ and $f_T(u)$ depend on the effective renormalized counterpart $u = u(\xi_-^{-1})$ of the bare coupling u_0. For $T \to T_\lambda(P)$, u approaches the fixed point value $u(0) = u^\star = 0.0362$. Our two-loop results read

$$f_\psi(u) = \frac{1}{32\pi u} + \left(-\frac{4}{27\pi} + \frac{2}{\pi}\ln 3\right) u + O(u^2)\,, \tag{7}$$

$$f_T(u) = 1 + \frac{8}{3}u + \left(\frac{464}{3} - 128\ln 3\right) u^2 + O(u^3)\,. \tag{8}$$

These functions are plotted in Fig. 1 in zero-, one- and two-loop order. Their fixed point values $f_\psi(u^\star)$ and $f_T(u^\star)$ determine the asymptotic amplitude A_ρ in (1) while their derivatives at $u = u^\star$ contribute to the subleading amplitude a_ρ.

While the two-loop correction to f_T is remarkably small (about 1%) at the fixed point, the two-loop contribution to $f_\psi(u)$ is about 10%, thus indicating a considerable uncertainty of low-order perturbation theory. The uncertainty is even larger for the derivative of $f_\psi(u)$ at $u^\star$ which implies a corresponding large uncertainty of a_ρ in (1).

A similar uncertainty exists with regard to the specific heat C^- below T_λ which is governed by the amplitude function $F_-(u)$ [3]. In two-loop order it reads

$$F_-(u) = \frac{1}{2u} - 4 + 64u + O(u^2)\,. \tag{9}$$

$F_-(u)$ is plotted in Fig. 2 in zero-, one and two-loop order. Its fixed point value and its derivative at $u = u^\star$ determine the aymptotic amplitude A^- and the subleading amplitude a_c^- in (2), respectively.

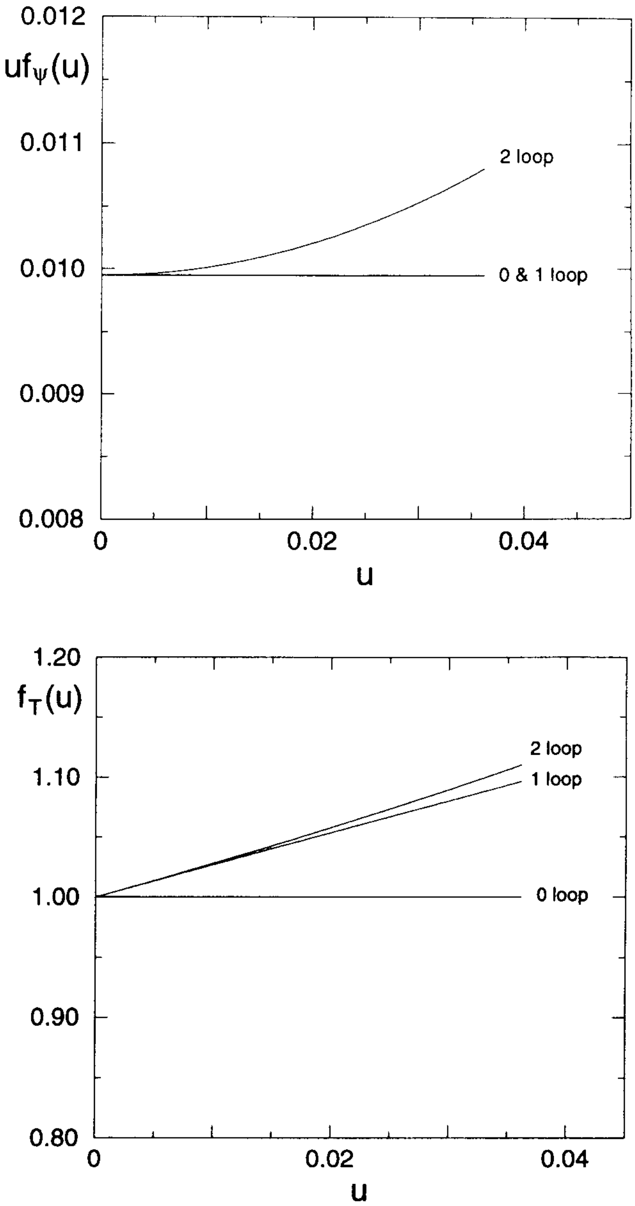

Fig. 1: Amplitude functions $uf_\psi(u)$ and $f_T(u)$ for the order parameter and for the k^2 part of the inverse of the transverse susceptibility in zero-, one- and two-loop order vs the renormalized coupling u. The curves terminate at the fixed point $u^\star = 0.0362$.

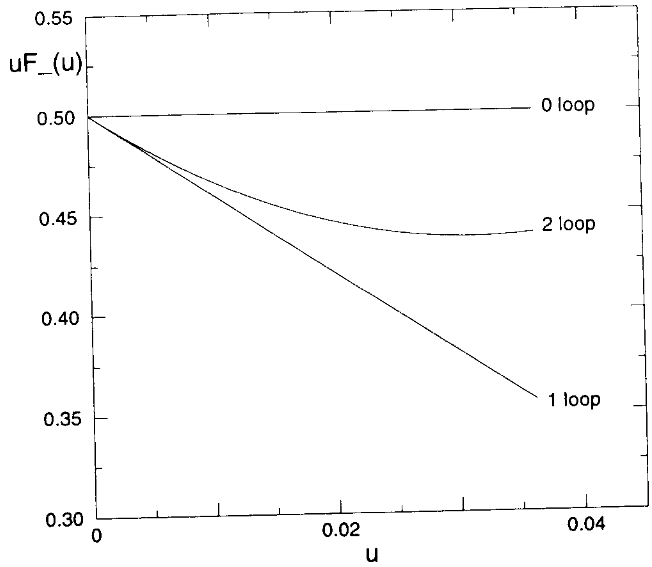

Fig. 2: Amplitude function $uF_{-}(u)$ for the specific heat below T_λ in zero-, one- and two-loop order vs the renormalized coupling u. The curves terminate at the fixed point $u^\star = 0.0362$.

In order to significantly reduce the uncertainty of the theoretical predictions on the superfluid density and the specific heat it is planned [4] to perform new higher-order field-theoretic RG calculations and Borel summations. Preliminary considerations [5] indicate that the amplitude functions of $|\langle\psi\rangle|^2$ and C^- can be determined up to four-loop order. Such amplitude functions contain the full information about the nonasymptotic critical behaviour which goes beyond the asymptotic representations (1), (2), (6) by including the entire Wegner series, i.e., higher-order terms $|t|^{n\Delta}$, $n = 2, 3, \ldots$ without introducing additional nonuniversal parameters [6].

Acknowledgement: We wish to thank DARA and JPL/NASA for support through contracts 50 WM 9669 and 960838.

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